SPECIALIA

A Note on the Relevancy of Behaviour of Solutions of the LANE-EMDEN Equation in the Immediate Neighbourhood of the Origin

In this note, the region in a complete polytrope to which the known behaviour of the solutions of the LANE-EMDEN equation, for different values of n, in the immediate neighbourhood of $\xi = 0$, will be relevant, has been pointed out.

The behaviour of solutions of the LANE-EMDEN equation near $\xi = 0$, for $n < 3$[1], $n = 3$[2] and $n > 3$[3] have been studied with a view to finding the arrangement of solutions near the origin of the polytropes; but recently, the author has shown[4] that in a polytrope, whatever be the index, solutions for n tending to zero and to minus one only are relevant. He has also shown[5] that the immediate neighbourhood of the origin is an interfacial region in which 2 sets of equations: one governing the origin and the other governing the rest of the parts of the configuration are relevant simultaneously. This gives an impression that probably solutions near $\xi = 0$ in $(\xi;\ \theta)$-plane, referred to above, have no physical meaning. The author wishes to point out that, although θ is not defined at the origin, θ is defined in the immediate neighbourhood of the interfacial region. The interfacial region is sufficiently close to the origin; therefore the known behaviour of solutions near $\xi = 0$ in $(\xi;\ \theta)$-plane, for different values of n, can be taken to be relevant to the immediate neighbourhood of the interfacial region of a polytrope[6].

Zusammenfassung. Untersuchungen über Lösungen der LANE-EMDEN-Gleichung in der Nähe des kritischen Punktes.

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[1] E. HOPF, Mon. Not. R. astr. Soc. *91*, 653 (1931).

[2] R. H. FOWLER, Mon. Not. R. astr. Soc. *91*, 63 (1930).

[3] S. CHANDRASHEKAHAR, *An Introduction to the Study of Stellar Structure* (University of Chicago Press 1939), p. 127.

[4] S. SRIVASTAVA, Experientia *24*, 319 (1968).

[5] S. SRIVASTAVA, Proc. natn. Acad. Sci. India, A, *36*, 909 (1966).

[6] The author is grateful to Dr. B. B. LAL, Professor and the Head of the department of Mathematics, K.N. Government College, Gyanpur, for the encouragement that he has given.

A Series Solution of the LANE-EMDEN Equation of Index 5

In this note, a series solution from which all classes of E-, F- and M-solutions of the LANE-EMDEN equation of index 5 can be derived, has been given.

A series solution of the LANE-EMDEN equation of index n, near the origin, giving E-solutions is known[1] but no such solution giving F- and M-solutions has yet been presented. The reason being that F- and M-solutions have a singularity at the origin; hence an expansion in the form of a series giving F- and M-solutions at the origin will not be possible.

In terms of z and t, defined as

$$z^2 = 2\,\vartheta^2\,\xi;\ \xi = e^{-t}, \tag{1}$$

LANE-EMDEN equation of index 5 after the first integration can be expressed as

$$\frac{dz}{dt} = \left[2D + \frac{z^2}{4} - \frac{z^6}{12}\right]^{1/2}. \tag{2}$$

$t = 0$ corresponds to $\xi = 1$ at which θ is finite for all classes of solutions. The homology theorem shows that we can take $z = 1$ at $t = 0$ without any loss of generality. At $t = 0$ therefore, we can assume a TAYLOR's expansion of the form

$$z = 1 + z_0^{(1)}\,t + \frac{1}{\underline{|2}}\,z_0^{(2)}\,t^2 + \frac{1}{\underline{|3}}\,z_0^{(3)}\,t^3 + \frac{1}{\underline{|4}}\,z_0^{(4)}\,t^4 + \ldots\ldots \tag{3}$$

where $z_0^{(n)}$ is the value of nth derivative at $t = 0$. Calculating different derivatives and substituting in (3), we get

$$z = 1 + \left(2D + \frac{1}{6}\right)^{1/2} t - \frac{1}{6}\left(2D + \frac{1}{6}\right)^{1/2} t^3 + \frac{5}{24}\left(2D + \frac{1}{6}\right) t^4 + \ldots\ldots \tag{4}$$

Reverting to $(\xi;\theta)$-variables, the above series can be written as

$$\vartheta = \frac{1}{(2\,\xi)^{1/2}}\left[1 - \left(2D + \frac{1}{6}\right)^{1/2} \log\xi + \frac{1}{6}\left(2D + \frac{1}{6}\right)^{1/2} (\log\xi)^3 - \frac{5}{24}\left(2D + \frac{1}{6}\right)(\log\xi)^4\right]. \tag{5}$$

If the series in (4) is terminated with t^4 then for the approximation to be good to 4 decimal places we must have[2]

$$\frac{5}{24}\left(2D + \frac{1}{6}\right) t^4 \leqq 0.00005\,. \tag{6}$$

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[1] S. CHANDRASHEKHAR, *An Introduction to the Study of Stellar Structure* (University of Chicago Press 1939), chap. 4.

[2] K. S. KUNZ, *Numerical Analysis* (McGraw-Hill Book Company, Inc., New York 1957), chap. 9.

Inequality in (6) suggests that for the series in (5) we must have

$$\xi \leqq e^{-[\cdot 00144/(12D+1)]^{1/4}} \tag{7}$$

Thus we see that, with the first 3 terms, the series in (5) gives θ to 4 decimal places over the range $1 < \xi < 1 + e^{[\cdot 00144/(12D+1)]^{1/4}}$ for any value of D.

By the method of continuing the solutions, one may continue the solution toward the centre as well as toward the boundary. By increasing the number of terms in (4), one may also increase the range sufficiently[3].

Zusammenfassung. Weitere Untersuchungen über Lösungen der Lane-Emden-Gleichung in der Nähe des kritischen Punktes (vgl. Experientia *23*, 697, 1967).

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Unit Cell Dimensions of Synthetic Apatites

The lattice parameters of some good synthetic apatites are reported here. The chlorapatite, fluorapatite, and mixed fluor-chlorapatites were kindly supplied by Dr. E. F. Apple, General Electric Company, Cleveland, Ohio, and calcium hydroxyapatite by Dr. W. E. Brown, National Bureau of Standards, Washington, D.C. The strontium hydroxyapatite was prepared by thoroughly mixing stoichiometric quantities of $Sr(NO_3)_2$, KH_2PO_4, and KOH according to the equation:

$$10\,Sr(NO_3)_2 + 6\,KH_2PO_4 + 14\,KOH \rightarrow Sr_{10}(PO_4)_6(OH)_2 + 20\,KNO_3 + 12\,H_2O$$

The unit cell dimensions were obtained by 2 different methods: in one, the X-ray traces were a plot of $2\,\theta$ along the chart drive direction versus intensity. The lowest $2\,\theta$ value was on the right and was usually 5°. The maximum scale reading for intensity was indicated in cps. The little red lines above the chart were degree markers and were used for reading accurate angles. The $2\,\theta$ angle was the angle between the incident X-ray beam and the diffracted beam. θ was the angle in Bragg's equation $\lambda = 2d_{hkl}\sin\theta$. CuK_α radiation was used. In the other method, the X-ray powder photographs were taken on a CGR instrument in a Seeman-Bolhin focussing camera using CoK_α radiation. The value 1.77892 Å was taken for the wave-length of this radiation. The powdered apatite and small amounts of NaCl (in case of calcium apatites) and calcite (in case of strontium hydroxyapatite) used as internal standard were ground and spread over a mount of polystyrene foil.

From hydroxyapatite to chlorapatite, *a* increases but *c* decreases. In case of fluorapatite, *a* decreases but *c* remains the same. Going on to mixed fluor-chlorapatites, *a* increases but *c* decreases. As the ionic radius of Sr^{+2} (1.13 Å) is greater than Ca^{+2} (0.99 Å), both *a* and *c* increase in case of strontium hydroxyapatite. The results are given in the Table.

The observed differences in lattice parameters are understandable from the various atomic positions in relation to the calcium triangle[1]. The hydroxide ions[2] of hydroxyapatite lie with their internuclear axes coincident with the six-fold screw axis. The larger *a* axis of hydroxyapatite is due to Ca-O distance which requires a larger calcium triangle[3]. The chloride ions of chlorapatite lie on the six-fold screw axes but midway between adjacent planes[1]. A model to explain the decrease in *c* and increase in *a* for chlorapatite has recently been presented[3]. The lattice difference between hydroxyapatite and fluorapatite is also brought about by change centered in the calcium triangle. Fluorine, being at the intersection of the planes with the six-fold screw axis, tends to contract the triangle until normal Ca-F distances are established[3].

Résumé. Les paramètres cristallographiques d'un certain nombre d'apatites synthétiques sont indiqués. A l'aide d'études par rayons X ces apatites synthétiques peuvent être distingués les unes des autres.

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Unit cell dimensions of synthetic apatites

Apatite	*a*-axis A	*c*-axis A	*c*/*a*
$Ca_{10}(PO_4)_6(OH)_2$	9.42	6.88	0.730
$Ca_{10}(PO_4)_6Cl_2$	9.63	6.78	0.704
$Ca_{10}(PO_4)_6F_2$	9.36	6.88	0.735
$Ca_{10}(PO_4)_6F_{0\cdot 554}Cl_{1\cdot 446}$	9.59	6.83	0.712
$Ca_{10}(PO_4)_6F_{1\cdot 449}Cl_{0\cdot 579}$	9.45	6.86	0.725
$Sr_{10}(PO_4)_6(OH)_2$	9.76	7.28	0.745

[1] J. M. Stutman, J. D. Termine and A. S. Posner, Trans. N.Y. Acad. Sci. *27*, 669 (1965).

[2] International Union of Pure and Applied Chemistry, Nomenclature of Inorganic Chemistry (1957). OH^- should be called 'hydroxide ion'.

[3] R. A. Young and J. C. Elliott, Archs oral Biol. *11*, 699 (1966).

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